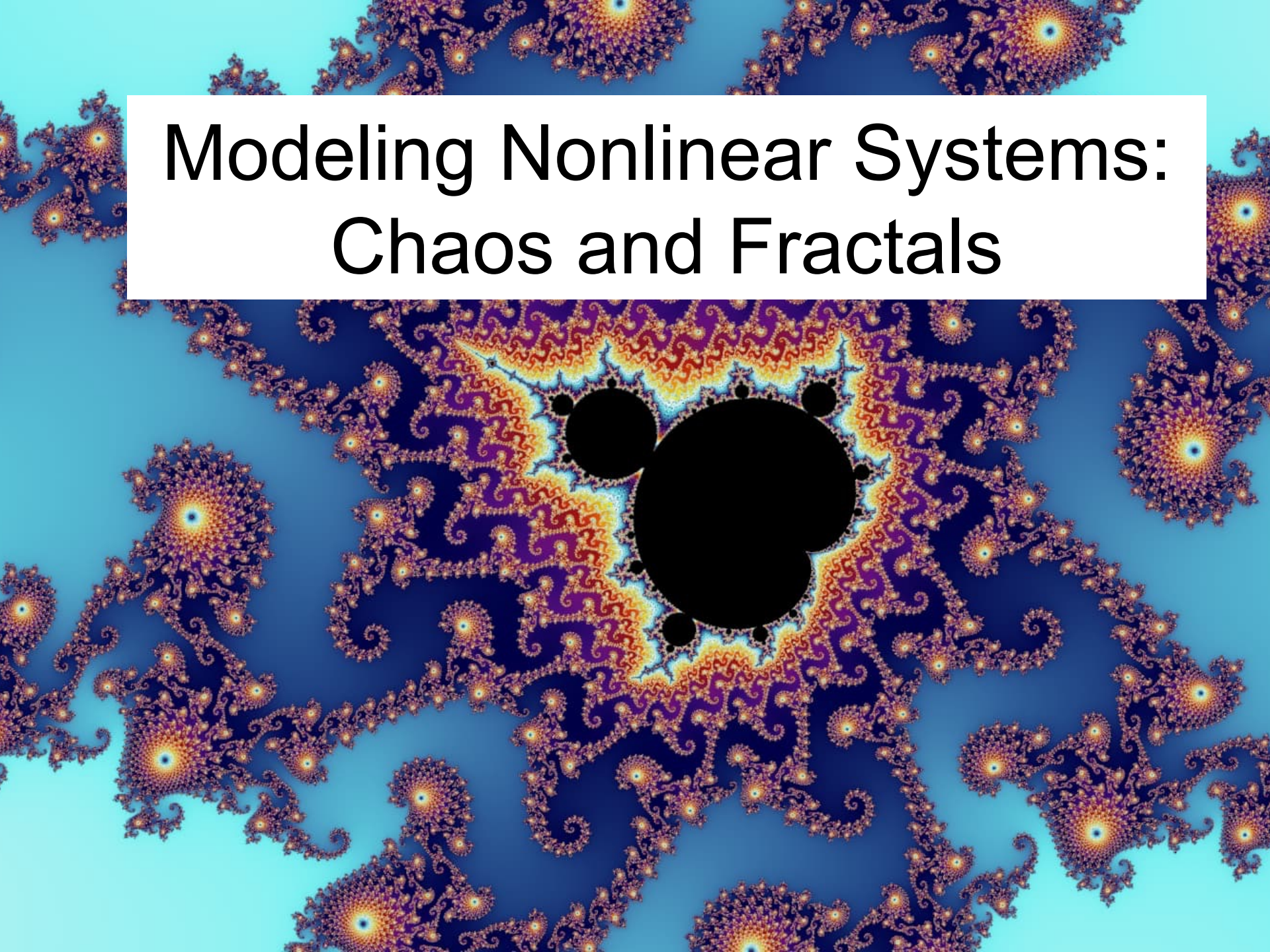


Modeling Nonlinear Systems: Chaos and Fractals



Nonlinear Dynamics: time evolution equations (e.g., Newton's 2nd law)/field equations contain nonlinear terms

There are many nonlinear systems in physics:

Mechanical systems: driven/coupled oscillators

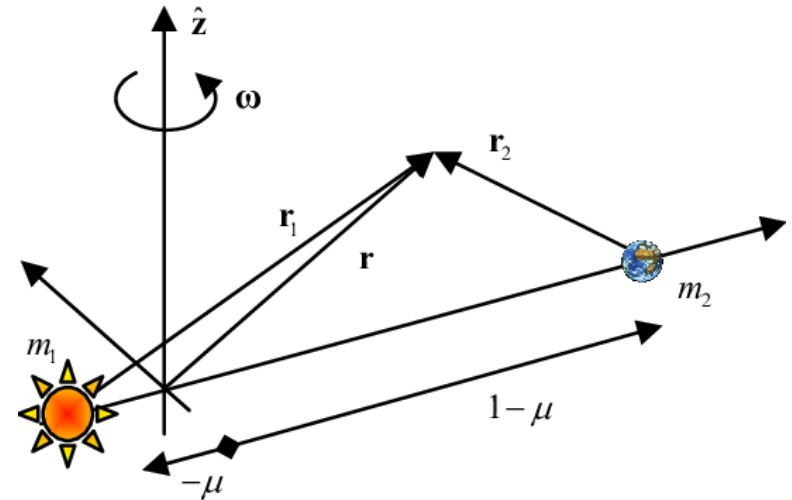
Astrophysics: **three-body problem**; stability of solar system; KAM theorem

General Relativity: Einstein equations

Electronics: nonlinear circuits

Hydrodynamics: turbulences

Climate science: weather patterns



Some nonlinear systems outside of physics :

Biology: predator-prey models

Chemistry: certain reactions...

Physiology: heart arrhythmias

Economics: stock market trends

Simple example: the **logistic equation** for population growth, modeled as a differential equation...

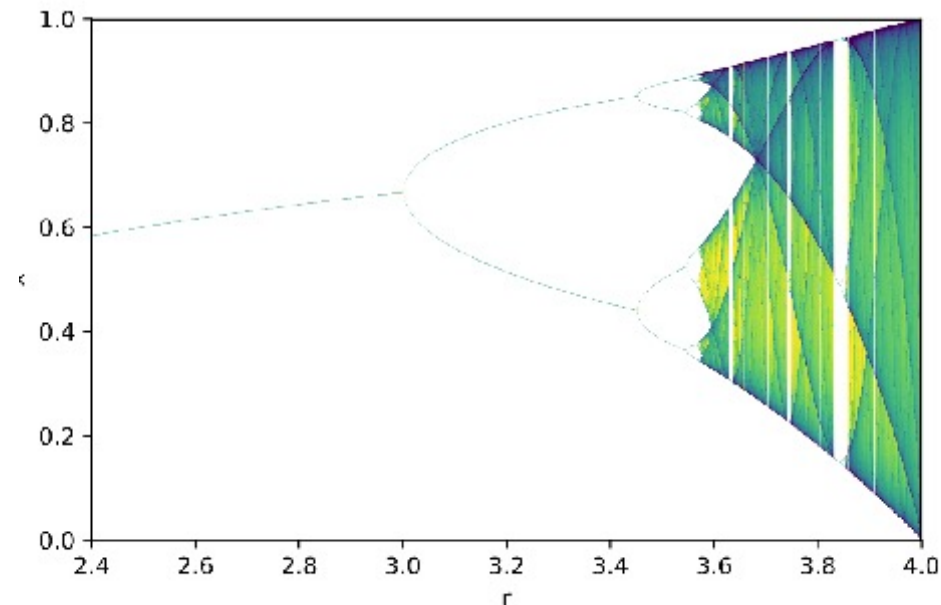
$$\frac{dx}{dt} = r x(1 - x)$$

...or as a difference equation...

$$x_{n+1} = r x_n(1 - x_n)$$

...graphed in Python:

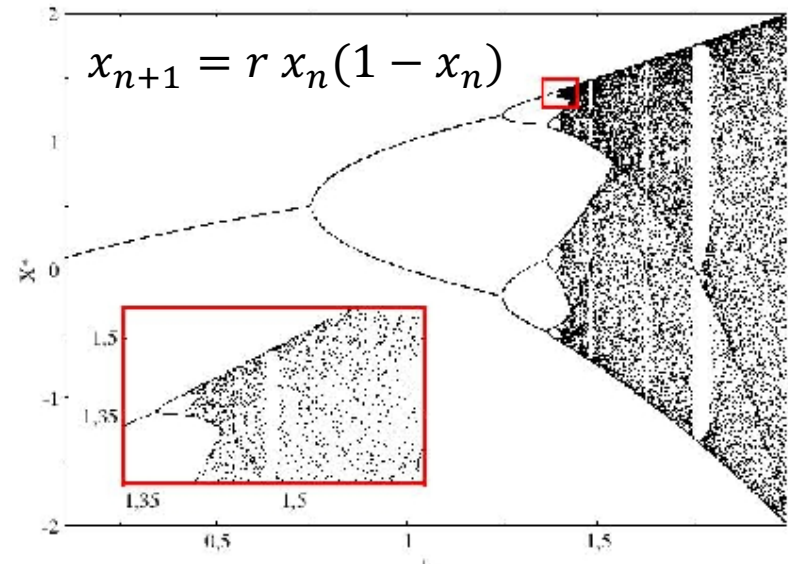
```
Editor - F:\PythonCode\vid\bifurcationdig.py
bifurcationdig.py
1#Bifurcation diagram (r vs x)
2#Logistic map: x=rx(1-x)
3import matplotlib.pyplot as plt
4import numpy as np
5R=np.linspace(2.5,4,10000) #control parameter range
6
7X = [] #X axis - r- control parameters
8Y = [] #Y axis - x values of the logistic map
9
10#Generate x for each value of r
11for r in R:
12    Xi.append(r)
13
14    x = np.random.random()#initialize x for each r value
15    for n in range(100): #ignore the transient effect
16        x=r*x*(1-x)
17
```



Bifurcations for $r > 3$, chaotic long-term behavior for $r \geq 3.57 \dots$,

Initial value problems (IVPs) usually have a unique solution; they are *deterministic*. Characteristic for nonlinear dynamical systems is their *Sensitive Dependence on Initial Conditions* (SDIC): similar initial conditions produce very different long-term behaviors. This is known as **deterministic chaos**.

The geometry of deterministic chaos often shows self-similar patterns: **fractals**.

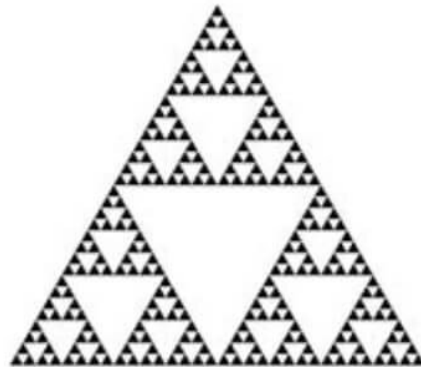


Fractals are everywhere:

In nature:



In geometry:



In algebra:

